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Top quark Kaluza-Klein mode mixing in the Randall-Sundrum bulk Standard Model and $B \to X_s\gamma$

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Abstract

We study the top quark Kaluza-Klein (KK) mode mixing in the Randall-Sundrum (RS) scenario, where all the standard model (SM) fields except the Higgs field are in the bulk. To avoid the problems of FCNC, for all three generations the universal bulk fermion mass m_ψ are assumed. The degeneracy in the KK masses of quarks leads to the GIM cancellation KK-level by level. However, Yukawa mass terms for the top quark generate the mixing among its KK modes. The degeneracy in the KK masses of up-type quarks is broken and their contribution to FCNC is inevitable. The current measurements of Br($B \to X_s + \gamma$) with 99% CL strongly constrain the effective weak scale k_{EW}: $k_{EW} \gtrsim 2$ TeV for $m_\psi/k = -0.3$ and $k_{EW} \gtrsim 3$ TeV for $m_\psi/k = -0.4$, where k is the AdS$_5$ curvature. Moreover, high energy collider and rare B decays can play complementary roles to probe the effects of the RS-bulk SM: The $m_\psi/k = -0.3$ ($m_\psi/k = -0.4$) case generates larger (smaller) collider signatures but smaller (larger) $b \to s\gamma$ contribution.

I. INTRODUCTION

Inspired by recent advances in string theories, particle physicists have resort to extra dimensions for the gauge hierarchy problem of the standard model (SM). Arkani-Hamed, Dimopoulos and Dvali (ADD) proposed that there exist n large extra dimensions with factorizable geometry [1]: The observed huge Planck scale $M_{\rm Pl}$ is attributed to the largeness of the extra dimension volume V_n, since $M_{\rm Pl}^2 = M_S^{n+2} V_n$ with M_S being the fundamental string scale. The hierarchy problem is resolved as M_S can be maintained around TeV. Later Randall and Sundrum (RS) proposed another higher dimensional scenario where the hierarchy problem is explained by a geometrical exponential factor, based on two branes and a single extra dimension with non-factorizable geometry [2]. In addition to solving the gauge hierarchy problem, TeV scale extra dimensions accessible to the SM fields have drawn a lot of interest due to various motivations such as gauge coupling unification [3], new mechanisms for supersymmetry breaking [4], the explanation of fermion mass hierarchies [5], and the presence of the Higgs doublet as a composite of top quarks [6].

Of great significance and interest is that extra dimensional models can lead to distinct and rich phenomenological signatures at near future colliders. In the ADD scenario, due to the large volume of extra dimensions, the SM fields should be confined to our brane. Only the gravity field, the fluctuation of space-time, propagates freely in extra dimensions, which are to be observed as Kaluza-Klein (KK) gravitons with almost continuous mass spectrum [7]. In the models of TeV scale extra dimensions accessible to the SM fields, KK excitations of the SM gauge (and possibly fermion) fields with TeV scale masses would be observed as resonances at future colliders [8,9]. In the original RS scenario, the SM fields are assumed to be confined to our brane while graviton fields are in the bulk: The KK gravitons have electroweak scale masses and couplings to matter, characterized by Λ_π. Small size of the RS extra dimension, however, phenomenologically allows that the SM fields are also in the bulk. In Ref. [10], it is demonstrated that placing the SM gauge fields in the RS-bulk while confining the fermions to our brane is strongly constrained by the current precision electroweak data so that the lowest KK state of gauge boson should be heavier than about 23 TeV. Then Λ_π should be pushed up to about 100 TeV, which is disfavored. If both the SM gauge and fermion fields are in the RS-bulk [11], their phenomenological signatures are sensitive to the bulk fermion mass m_ψ, which determines the KK mass spectrum of the bulk fermions and their interactions with bulk gauge bosons and other bulk fermions.

It is well known that any kind of new physics beyond the SM is significantly constrained by flavor changing neutral current (FCNC), which the SM predicts to be suppressed at one loop level by light quark masses relative to M_W and by small CKM mixing between the third and first two generations. In the RS-bulk SM, a simple way to avoid the problems of FCNC as well as of proton decay is suggested by assuming the universal bulk fermion mass for all three generation quarks [11,12]: The KK excitation mass spectra are the same for up, charm and top quarks. With the minimal flavor assumption that at the tree level the flavor mixing comes only from the CKM matrix, the GIM cancellation [13] occurs KK-level by level. However, there is an inevitable source of non-degeneracy in the bulk fermion masses from Yukawa interaction with the Higgs fields. As pointed out in Refs. [11,14], the Yukawa interaction mixes the fermion KK tower members, which can be substantial for the top quark. Recently it has been shown that the universality of the top quark gauge coupling can deviate from the SM prediction by up to 4% if the product of the five-dimensional Yukawa coupling and the AdS$_5$ curvature k is below 10 [15].

In this paper we study in detail the top quark KK mode mixing in the RS-bulk SM, and its effects on the rare decay of $B \to X_s\gamma$. We notice that this mixing breaks the degeneracy in the mass of the KK excited modes of up-type quarks at a given level. One of the most important consequences is the non-vanishing FCNC. Through a careful analysis, we demonstrate that this mixing is very sensitive to m_ψ and can be quite large in parts of the parameter space of $\nu \equiv m_\psi/k$. Moreover, our full numerical estimation shows that for a given ν, the behavior of top quark KK mixing effects on $b \to s\gamma$ is opposite to that on the high energy collider signatures; in the parameter space where the mixing contribution to FCNC is large, its phenomenological signatures at colliders are small and vice versa. Thus, high energy collider and rare B decays can play complementary roles to probe the effects of the RS-bulk SM.

This paper is organized as follows. In Sec. II, we briefly review the RS-bulk SM, focused on the KK reduction and interactions of bulk gauge fields and bulk fermions. In particular, we point out a subtle point when accommodating the SM fermion sector in the RS-bulk: The fermion field contents should be doubled. In Sec. III, the mass matrix of the KK modes of a bulk fermion is given, including the Yukawa interactions with the Higgs field confined to our brane. Mixing among the KK modes of the top quark occurs. In the part of parameter space where the elements of the Yukawa mass matrix are smaller than those of the KK mass matrix, the diagonalization can be made perturbatively. In Sec. IV, we discuss the effects of the KK modes of W gauge bosons and up-type quarks on the decay rate of $b \to s\gamma$. After presenting a general expression of the modified Wilson coefficient, we show that in the parameter space, where perturbative diagonalization is feasible, the effects of the RS-bulk SM vanish to leading order. And in Sec. V we present our numerical results, thus demonstrating that the observed decay rate of $b \to s\gamma$ imposes quite a strong lower bound on the effective electroweak scale $k_{EW}(\equiv e^{-kr_c\pi}k)$. Finally, Sec. VI represents summary and conclusions.

II. BULK SM IN THE RS SCENARIO

In the RS scenario, a single extra dimension has been proposed with non-factorizable geometry, which is compactified on a S_1/Z_2 orbifold of radius r_c [2]. Requiring four-dimensional Poincaré invariance, the RS configuration has the following solution to the five-dimensional Einstein's equation:

$$ds^2 = G_{MN}dx^M dx^N = e^{-2\sigma(\phi)}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2, \tag{1}$$

where the four-dimensional metric tensor is defined by $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$, $\sigma(\phi) \equiv kr_c|\phi|$, $0 \leq |\phi| \leq \pi$, and the five-dimensional curvature is $R_5 = -20\,k^2$. Upper case Roman indices run over all the five dimensions while the Greek indices over our four dimensions. Two orbifold fixed points accommodate two three-branes, the Planck brane at $\phi = 0$ and our TeV brane at $|\phi| = \pi$. Due to the assignment of our brane at $|\phi| = \pi$, a fundamental scale m_0 of the order Planck scale appears gravitationally red-shifted by the so-called warp factor of $\epsilon \equiv e^{-kr_c\pi}$. If $kr_c \approx 12$, the hierarchy problem can be resolved. From the four-dimensional effective action, the relation between four-dimensional Planck scale $M_{\rm Pl}$ and the fundamental string scale M_5 is obtained by $M_{\rm Pl}^2 = M_5^3(1 - e^{-2kr_c\pi})/k$. Now let us review the masses and couplings of the KK modes of the bulk gauge and fermion fields, which are relevant to the $b \to s\gamma$ decay. The back-reaction of the bulk fields on the AdS metric is to be neglected.

A. Gauge field KK spectrum

For a massless $SU(2)$ gauge field $A_M^a(x,\phi)$, the gauge invariant five-dimensional action is [10,16]

$$S_A = -\frac{1}{4}\int d^5x\sqrt{-G}\,G^{MK}G^{NL}F_{KL}^a F_{MN}^a, \tag{2}$$

where $F_{MN}^a = \partial_M A_N^a - \partial_N A_M^a - g_5\epsilon^{abc}A_M^a A_N^b (a,b,c = 1,2,3)$. By choosing the odd Z_2-parity for $A_5^a(x,\phi)$ [10] and/or an appropriate gauge [17], $A_5^a(x,\phi)$ decouples from the Lagrangian. With the KK expansion of

$$A_\mu^a(x,\phi) = \sum_{n=0}^{\infty} A_\mu^{a(n)}(x)\frac{\chi_A^{(n)}(\phi)}{\sqrt{r_c}}, \tag{3}$$

we have the four-dimensional effective action of massive KK gauge bosons such as

$$S_A = \int d^4x \sum_{n=0}^{\infty}\left[-\frac{1}{4}\eta^{\mu\kappa}\eta^{\nu\lambda}F_{\kappa\lambda}^{a(n)}F_{\mu\nu}^{a(n)} - \frac{M_A^{(n)\,2}}{2}\eta^{\mu\nu}A_\mu^{a(n)}A_\nu^{a(n)}\right], \tag{4}$$

which is obtained by

$$\chi_A^{(n)}(\phi) = \frac{e^{\sigma(\phi)}}{N_A^{(n)}}\left[J_1(z_A^{(n)}(\phi)) + \alpha_A^{(n)}Y_1(z_A^{(n)}(\phi))\right]. \tag{5}$$

Here, $z_A^{(n)}(\phi)$ and the normalization $N_A^{(n)}$ are given by

$$z_A^{(n)}(\phi) = \frac{M_A^{(n)}}{k} e^{\sigma(\phi)}, \tag{6}$$
$$N_A^{(n)} = \frac{e^{kr_c\pi}}{\sqrt{kr_c}} \left| J_1(x_A^{(n)}) \right|,$$

where $x_A^{(n)} \equiv z_A^{(n)}(\pi) = M_A^{(n)}/k_{EW}$ with $k_{EW} \equiv \epsilon\, k$. Note that the wave function $\chi_A^{(n)}(\phi)$ satisfies the orthonormal condition

$$\int_{-\pi}^{\pi} d\phi\, \chi_A^{(m)}(\phi)\chi_A^{(n)}(\phi) = \delta^{mn}, \tag{7}$$

which leads to $\chi_A^{(0)} = 1/\sqrt{2\pi}$. The continuity of $d\chi_A^{(n)}/d\phi$ at $\phi = 0$ and $\phi = \pm\pi$ determines the coefficient $\alpha_A^{(n)}$ to be

$$\alpha_A^{(n)} = -\frac{J_0(M_A^{(n)}/k)}{Y_0(M_A^{(n)}/k)}, \tag{8}$$

and $x_A^{(n)}$ to be the roots of the following equation:

$$J_0(x_A^{(n)}) + \alpha_A^{(n)} Y_0(x_A^{(n)}) = 0. \tag{9}$$

Numerically we have $x_A^{(1)} \approx 2.45$, $x_A^{(2)} \approx 5.57$, $x_A^{(3)} \approx 8.70$, and $x_A^{(4)} \approx 11.84$.

B. Review of Grossman-Neubert fermion KK spectrum

Let us review in detail the KK solutions of a bulk fermion with arbitrary Dirac bulk mass in the RS scenario [12,18,19], which causes a subtle problem when discussing the bulk SM. The five-dimensional action of a Dirac fermion Ψ with the bulk mass m_ψ is

$$S = \int \mathrm{d}^4x \int \mathrm{d}\phi \sqrt{-G} \left\{ E_{\underline{A}}^A \left[\frac{i}{2}\, \bar{\Psi}\gamma^{\underline{A}}(D_A - \overleftarrow{D_A})\Psi + \frac{1}{8}\, \omega_{\underline{BC}A}\, \bar{\Psi}\{\gamma^{\underline{A}}, \sigma^{\underline{BC}}\}\Psi \right] \right. \tag{10}$$
$$\left. - m_\psi \, \mathrm{sign}(\phi)\, \bar{\Psi}\Psi \right\},$$

where D_A is the covariant derivative, and the underlined upper case Roman indices describe objects in the tangent frame. Further, $\gamma^{\underline{A}} = (\gamma^\mu, i\gamma_5)$ and the inverse vielbein $E_{\underline{A}}^A = \mathrm{diag}(e^\sigma, e^\sigma, e^\sigma, e^\sigma, 1/r_c)$, and the contribution of the spin connection $\omega_{\underline{BC}A}$ vanishes by including the hermitian conjugates.

Integration by parts leads to the action

$$S = \int \mathrm{d}^4x \int \mathrm{d}\phi\, r_c \Big\{ e^{-3\sigma} \left(\bar\Psi_L\, i\partial\!\!\!/\, \Psi_L + \bar\Psi_R\, i\partial\!\!\!/\, \Psi_R \right) \\ - \frac{1}{2r_c} \left[\bar\Psi_L \left(e^{-4\sigma}\partial_\phi + \partial_\phi\, e^{-4\sigma} \right) \Psi_R - \bar\Psi_R \left(e^{-4\sigma}\partial_\phi + \partial_\phi\, e^{-4\sigma} \right) \Psi_L \right] \\ - e^{-4\sigma}\, m_\psi\, \mathrm{sign}(\phi) \left(\bar\Psi_L \Psi_R + \bar\Psi_R \Psi_L \right) \Big\}, \tag{11}$$

where we impose periodic boundary conditions $\Psi_{L,R}(x,\pi) = \Psi_{L,R}(x,-\pi)$. With the KK expansion of Ψ

$$\Psi_{L,R}(x,\phi) = \sum_{n=0}^{\infty} \psi^{(n)}_{L,R}(x)\, \frac{e^{2\sigma(\phi)}}{\sqrt{r_c}}\, \hat f^{(n)}_{L,R}(\phi)\,, \tag{12}$$

and the requirement of

$$\int\limits_{-\pi}^{\pi} \mathrm{d}\phi\, e^{\sigma} \hat f^{(m)*}_L(\phi)\, \hat f^{(n)}_L(\phi) = \int\limits_{-\pi}^{\pi} \mathrm{d}\phi\, e^{\sigma} \hat f^{(m)*}_R(\phi)\, \hat f^{(n)}_R(\phi) = \delta^{mn}\,, \tag{13}$$

$$\left(\pm \frac{1}{r_c}\, \partial_\phi - m \right) \hat f^{(n)}_{L,R}(\phi) = -M^{(n)}_f\, e^{\sigma}\, \hat f^{(n)}_{R,L}(\phi)\,, \tag{14}$$

we have a tower of massive Dirac fermions with the effective action

$$S = \sum_{n=0}^{\infty} \int \mathrm{d}^4x \left\{ \bar\psi^{(n)}(x)\, i\partial\!\!\!/\, \psi^{(n)}(x) - M^{(n)}_f\, \bar\psi^{(n)}(x)\, \psi^{(n)}(x) \right\}. \tag{15}$$

Note that Z_2-symmetric action constrains $\bar\Psi\Psi = \bar\Psi_L\Psi_R + \bar\Psi_R\Psi_L$ ($\Psi_{L,R} \equiv (1 \mp \gamma_5)\Psi/2$) to be Z_2-odd, as can be seen from the first term in Eq. (10). If $f^{(n)}_L$ is a Z_2-even function $\chi^{(n)}$ then $f^{(n)}_R$ should be a Z_2-odd function $\tau^{(n)}$ and vice versa. With $\nu \equiv m_\psi/k$ of the order unity, the solutions are, for $n \neq 0$,

$$\chi^{(n)}(\phi) = \frac{e^{\sigma/2}}{N^{(n)}_\chi} \left[J_{1/2-\nu}(z^{(n)}) + \beta^{(n)}_\chi Y_{1/2-\nu}(z^{(n)}) \right], \tag{16}$$

$$\tau^{(n)}(\phi) = \frac{e^{\sigma/2}}{N^{(n)}_\tau} \left[J_{1/2+\nu}(z^{(n)}) + \beta^{(n)}_\tau Y_{1/2+\nu}(z^{(n)}) \right],$$

and for $n = 0$

$$\chi^{(0)}(\phi) = \frac{e^{\nu\sigma(\phi)}}{N^{(0)}_\chi}, \quad \tau^{(0)}(\phi) = 0. \tag{17}$$

With the even Z_2-parity of $\chi^{(n)}$ and the odd parity of $\tau^{(n)}$, Eq. (14) yields boundary conditions

$$0 = \left(\frac{d}{d\phi} - mr_c\right) \chi^{(n)}\Big|_{\phi=0,\pi} = \tau^{(n)}\Big|_{\phi=0,\pi} , \tag{18}$$

which determine the coefficients $\beta_{\chi,\tau}^{(n)}$ as well as the KK fermion mass $M_f^{(n)} \equiv x_f^{(n)} k_{EW}$. Since only Z_2-even KK modes contribute to $b \to s\gamma$ decay as shall be discussed later, we present the expressions for the Z_2-even part:

$$\beta_\chi^{(n)} = -\frac{J_{-(\nu+1/2)}(M_f^{(n)}/k)}{Y_{-(\nu+1/2)}(M_f^{(n)}/k)} , \tag{19}$$

$$N_\chi^{(n)} = \frac{e^{kr_c\pi}}{x_f^{(n)}\sqrt{kr_c}} \sqrt{ z_\chi^{(n)2} \left\{ J_{\frac{1}{2}-\nu}(z_\chi^{(n)}) + \beta_\chi^{(n)} Y_{\frac{1}{2}-\nu}(z_\chi^{(n)}) \right\}^2 \Big|_{\phi=0}^{\phi=\pi} } , \tag{20}$$

where

$$N_\chi^{(0)} = \frac{1}{\epsilon^{(\nu+1/2)}} \sqrt{\frac{2}{kr_c} \left| \frac{1-\epsilon^{1+2\nu}}{1+2\nu} \right|} . \tag{21}$$

And $x_f^{(n)}$ is the solution of

$$J_{-(\nu+1/2)}(x_f^{(n)}) + \beta_\chi^{(n)} Y_{-(\nu+1/2)}(x_f^{(n)}) = 0 , \tag{22}$$

which is the same as that of the right-handed one. We refer to Ref. [11] for the corresponding expressions of the Z_2-odd part.

Discussions on the physical and phenomenological implications of the parameter ν are in order here. Note that the canonically re-scaled zero mode of Z_2-even bulk fermion is proportional to $e^{(1/2+\nu)kr_c|\phi|}$ (see the first line of Eq. (11) with Eqs. (12) and (17)). For $\nu \ll -1/2$ the fermion bulk wave functions are localized toward the Planck brane: The magnitudes of its gauge couplings with KK gauge bosons are quite small. In Refs. [12,20], it has been numerically demonstrated that the couplings are small for $\nu \lesssim -0.5$ and thus it is difficult to probe the RS-bulk SM effects at high energy colliders. However, as shall be shown below, this parameter space of $\nu \lesssim -0.5$ induces large mixing in the mass matrix of the KK excited modes of top quark, resulting in substantial contribution to FCNC such as $b \to s\gamma$. For $\nu \gg 1$ the SM fermions become localized closer to the TeV brane: The model approaches the RS model with only the gauge fields in the bulk, which is phenomenologically disfavored due to unreasonably large $M_A^{(1)}$, the mass of the first excited KK gauge boson [10]. To be specific, if $\nu \gtrsim -0.3$, the large contributions to the precision electroweak data push $M_A^{(1)}$ up to about 6 TeV, beyond the direct production at any planned collider. In what follows, therefore, we consider the parameter space of ν between -0.5 and -0.3.

C. Bulk SM in the RS scenario

In order to place the SM fermions in the bulk of the RS background, the fermion field contents should be doubled. In the SM, a fermion field with left-handed chirality and that with right-handed chirality belong to different representations of a gauge group: For example, the left-handed up quark (u_L) and the left-handed down quark (d_L) form an $SU(2)$-doublet, while the right-handed up and down quarks (u_R and d_R) are two $SU(2)$-singlet fields. u_L and u_R are related by the Dirac mass term, Yukawa coupling with the Higgs field. Two Dirac fermion fields, u and d, are enough to describe each generation in the quark sector. In the RS-bulk SM, a particle which belongs to a given representation of a gauge group should possess both left- and right-handed chiralities. If only one chiral state (e.g., Ψ_L) exists, the second line in Eq. (11) as well as the bulk mass term vanish: Non-trivial solution of the bulk function $f^{(n)}(\phi)$ cannot be obtained.

For each generation, we introduce four five-dimensional Dirac fields, an SU(2)-doublet fermion field $Q = (q_u, q_d)$ and two SU(2)-singlet fermion fields, u and d, with weak hypercharges $Y = -1/2$ and -1, respectively. The five dimensional fermion action is then

$$S = \int \mathrm{d}^4x \int \mathrm{d}\phi \, \sqrt{-G} \left\{ E_a^A \left[i\, \bar{Q}\gamma^a \mathcal{D}_A Q + i\, \bar{u}\gamma^a \mathcal{D}_A u + i\, \bar{d}\gamma^a \mathcal{D}_A d \right] + h.c. \right. \\ \left. - m_\psi \, \mathrm{sign}(\phi)\, (\bar{Q}Q + \bar{u}u + \bar{d}d) \right\} . \tag{23}$$

We assume that all the bulk masses are the same to avoid the large FCNC. The KK reduction leads to the four-dimensional Lagrangian of corresponding KK towers such that

$$\mathcal{L} = -\sum_{n=1}^{\infty} M_f^{(n)} \left[\bar{q}_{uL}^{(n)} q_{uR} + \bar{u}_L^{(n)} u_R^{(n)} \right] + h.c.. \tag{24}$$

Note that the KK masses are the couplings of $\bar{q}_u q_u$, not of $\bar{q}_u u$.

Since the SM fermions should correspond to the KK zero modes, we assign Z_2-even wave function $\chi^{(n)}$ to the left-handed $SU(2)$ doublet and the right-handed $SU(2)$-singlet such that

$$Q(x,\phi) = Q_L + Q_R = \sum_n \frac{e^{2\sigma(\phi)}}{\sqrt{r_c}} \left[Q_L^{(n)}(x)\chi^{(n)}(\phi) + Q_R^{(n)}(x)\tau^{(n)}(\phi) \right], \tag{25}$$
$$u(x,\phi) = u_L + u_R = \sum_n \frac{e^{2\sigma(\phi)}}{\sqrt{r_c}} \left[u_L^{(n)}(x)\tau^{(n)}(\phi) + u_R^{(n)}(x)\chi^{(n)}(\phi) \right],$$

and $d(x,\phi)$ has the same KK decomposition as $u(x,\phi)$.

The charged current interactions, mediated by the bulk W boson, connect q_u and q_d:

$$S_{f\bar{f}'W^\pm} = \int d^5x \sqrt{-G} e^\sigma \frac{g_5}{\sqrt{2}} \left[\bar{q}_u \not{W}^+ q_d + h.c. \right] \tag{26}$$
$$= \int d^4x \frac{g}{\sqrt{2}} \sum_{l=0}^{\infty} \left[\sum_{n,m=0}^{\infty} \bar{q}_{uL}^{(n)} \not{W}^{+(l)} q_{dL}^{(m)} \left\{ C_{nml}^{\bar{f}f'W} \right\} \right. \tag{27}$$
$$\left. + \sum_{n,m=1}^{\infty} \bar{q}_{uR}^{(n)} \not{W}^{+(l)} q_{dR}^{(m)} \left\{ \sqrt{2\pi} \int_{-\pi}^{\pi} d\phi\, e^\sigma \tau^{(n)} \tau^{(m)} \chi_A^{(l)} \right\} \right] + h.c.,$$

where $g = g_5/\sqrt{2\pi r_c}$, and the KK expansion in Eqs. (3) and (25) have been substituted. $C_{nml}^{\bar{f}f'W}$ denotes the coupling of the m-th and the n-th fermion states to the l-th W boson in the unit of the SM coupling. It is defined by

$$C_{nml}^{\bar{f}f'W} = \sqrt{2\pi} \int_{-\pi}^{\pi} d\phi\, e^\sigma \chi^{(n)}(\phi) \chi^{(m)}(\phi) \chi_A^{(l)}(\phi). \tag{28}$$

III. MASS MATRIX OF THE KK FERMION MODES

In addition to the KK masses, an observer on the TeV brane has another source for the fermion mass, Yukawa coupling with the Higgs boson. The Higgs mechanism should operate here so that the KK zero modes of the bulk gauge bosons and fermions, which correspond to the SM particles, acquire masses. In the simplest case where the Higgs field is also in the bulk, however, there are some unsatisfactory consequences. First, the lowest mass eigenvalue of the gauge boson, proportional to the bulk mass of the Higgs field, has no suppression by the warp factor [19]. The Higgs bulk mass should be much smaller than the Planck mass scale. The gauge hierarchy problem has recurred. Second, the bulk Higgs mechanism cannot retain the correct SM gauge couplings of the photon, W and Z bosons for the SM fermions off the TeV brane; if the fermions are on the wall, the SM mass relationship of the W and Z bosons is broken [11]. It is concluded that at least one Higgs doublet field must be confined to the TeV brane.

The five-dimensional action for Yukawa interaction with the confined Higgs field is

$$S_{ffH} = -\frac{\lambda_5}{k}\int d^5x\sqrt{-G}\Big[\bar{Q}(x,\phi)\cdot H(x)d(x,\phi) \\ + \epsilon^{ab}\bar{Q}(x,\phi)_a\cdot H(x)_b u(x,\phi) + h.c.\Big]\,\delta(\phi-\pi)\,, \tag{29}$$

where λ_5 is the five-dimensional Yukawa coupling. Here the quark fields are rotated so that their Yukawa matrix becomes diagonal in the quark generation space. The mismatch between the mass eigenstates and the electroweak eigenstates at the tree level is assumed to be the same as in the SM; minimal flavor mixing is assumed such that the CKM matrix is the only source for generation mixing at tree level. Spontaneous symmetry breaking shifts the Higgs field as $H^0 \to v_5 + H'^0$ with a VEV of the order Planck scale. Then the four-dimensional effective Lagrangian becomes

$$\mathcal{L}_{eff} = \frac{\lambda v_4}{\sqrt{2}}\left(\bar{q}_{uL}^{(0)} + \hat{\chi}_1\bar{q}_{uL}^{(1)} + \cdots\right)\left(u_R^{(0)} + \hat{\chi}_1 u_R^{(1)} + \cdots\right)\,, \tag{30}$$

where $\lambda = \lambda_5(1+2\nu)/2(1-\epsilon^{1+2\nu})$, $v = \epsilon v_5$, and $\hat{\chi}_n \equiv \chi^{(n)}(\pi)/\chi^{(0)}$. Note that $q_{uR}^{(n)}$ and $u_L^{(n)}$ do not appear in the Yukawa term due to the boundary conditions for τ in Eq. (18).

For definite presentation, we introduce the number of the KK states, n_∞, which is in principle infinity. Equations (24) and (30) imply the mass term of the KK modes of a bulk fermion as follows:

$$\mathcal{L}_{mass} = -\left(\bar{u}_R^{(0)}\bar{u}_R^{(1)}\cdots|\bar{q}_{uR}^{(1)}\cdots\right)\mathcal{M}_q\begin{pmatrix} q_{uL}^{(0)} \\ q_{uL}^{(1)} \\ \vdots \\ \hline u_L^{(1)} \\ \vdots \end{pmatrix}\,, \tag{31}$$

where the $(2n_\infty+1)\times(2n_\infty+1)$ matrix $\mathcal{M}_q$ is defined by

$$\mathcal{M}_q = \begin{pmatrix} \mathcal{M}_Y & \mathcal{M}_{KK} \\ \mathcal{M}_{KK}^\dagger & 0 \end{pmatrix}. \tag{32}$$

The $(n_\infty+1)\times(n_\infty+1)$ matrix $\mathcal{M}_Y$ is from Yukawa mass terms, and the $(n_\infty+1)\times n_\infty$ matrix $\mathcal{M}_{KK}$ from KK masses. They are given by

$$\mathcal{M}_Y = m_0 \begin{pmatrix} 1 & \hat{\chi}_1 & \hat{\chi}_2 & \cdots \\ \hat{\chi}_1 & \hat{\chi}_1^2 & \hat{\chi}_1\hat{\chi}_2 & \cdots \\ \hat{\chi}_2 & \hat{\chi}_2\hat{\chi}_1 & \hat{\chi}_2^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad \mathcal{M}_{KK} = k_{EW} \begin{pmatrix} 0 & 0 & \cdots \\ x_f^{(1)} & 0 & \cdots \\ 0 & x_f^{(2)} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}, \tag{33}$$

where $m_0 = \lambda v_4/\sqrt{2}$ of electroweak scale. Note that the bottom-right block of $\mathcal{M}_q$ vanishes since there are no Yukawa couplings of bulk fermions with the opposite chirality to the SM fermions.

A real and symmetric matrix $\mathcal{M}_q$ is diagonalized by an orthogonal matrix $\mathcal{N}$:

$$\mathrm{diag}(\eta_0,\eta_1,\cdots)\mathcal{N}\mathcal{M}_q\mathcal{N}^T = \mathrm{diag}(m_q, M_1, M_2, \cdots), \tag{34}$$

where $\eta_{j'} = \pm 1$ is introduced for the positive-definite mass. $q_{uL}^{(n)}$ and $u_L^{(n)}$ form $(2\,n_\infty+1)$ left-handed mass eigenstates ${u'_L}^{(\underline{j})}$

$$\begin{pmatrix} {u'_L}^{(0)} \\ {u'_L}^{(1)} \\ {u'_L}^{(2)} \\ \vdots \end{pmatrix} = \mathcal{N} \begin{pmatrix} q_{uL}^{(0)} \\ q_{uL}^{(1)} \\ \vdots \\ \hline u_L^{(1)} \\ \vdots \end{pmatrix}. \tag{35}$$

Since the charged current of our interest includes at least one SM fermion (the zero mode), only $q_{uL}^{(n)}$ becomes involved, which is a mixture of KK mass eigenstates ${u'}_L^{(\underline{j})}$

$$q_{uL}^{(i)} = \sum_{j'=0}^{2n_\infty} \mathcal{N}_{(\underline{j},\,i)} {u'_L}^{(\underline{j})}, \quad (i = 0, \cdots, n_C)\,. \tag{36}$$

In what follows, the underlined index runs from zero to $2\,n_\infty$. In terms of the mass eigenstates, the four-dimensional effective Lagrangian relevant for FCNC is

$$\mathcal{L} = \frac{g}{\sqrt{2}} V_{tb} \left(\sum_{m=0}^{n_\infty} C_{0ml}^{btW} \mathcal{N}_{(\underline{j},\,m)} \right) \bar{b}_L^{(0)} \gamma^\mu t_L^{\prime(\underline{j})} W_\mu^{(l)} + h.c.. \tag{37}$$

Now let us discuss the diagonalization of $\mathcal{M}_q$. For light quarks ($m_0 = 0$), $\mathcal{M}_q$ can be analytically diagonalized. Mass eigenvalues are

$$m_q^{(0)} = 0, \quad M_i^{(0)} = M_{n_\infty+i}^{(0)} = x_f^{(i)} k_{EW}, \tag{38}$$

which are obtained by the orthonormal matrix $\mathcal{N}^{(0)}$:

$$\mathcal{N}_{(0,\,0)}^{(0)} = 1, \quad \mathcal{N}_{(0,\,\underline{i})}^{(0)} = \mathcal{N}_{(\underline{i},\,0)}^{(0)} = 0, \quad \mathcal{N}_{(i,\,j)}^{(0)} = -\mathcal{N}_{(n_\infty+i,\,j)}^{(0)} = \frac{\delta_{ij}}{\sqrt{2}}. \tag{39}$$

For top quark with non-negligible m_0, the diagonalization of $\mathcal{M}_q$ is not trivial. Since m_0 is approximately the top quark mass of 175 GeV while the KK fermion masses are of the order TeV, the diagonalization can be made perturbatively unless $\hat{\chi}_n^2$'s are much larger than unity. In Fig. 1 we present the values of $\hat{\chi}_n$ which are very sensitive to ν: The solid line is for $\hat{\chi}_{2n-1}$ and the dashed line is for $\hat{\chi}_{2n}$ ($n = 1, 2, \cdots$). Note that $\hat{\chi}^{(n)}$ does not depend on k_{EW}. It can be seen that, for $\nu \lesssim -0.4$, $|\hat{\chi}_n|$ rapidly increases so that the elements of $\mathcal{M}_Y$ can be compatible or even larger than some elements of $\mathcal{M}_{KK}$: Diagonalization is to be made only numerically.

For $\nu \gtrsim -0.3$, $\delta (\equiv m_0/k_{EW})$ is a good perturbation parameter. To leading order in δ, the mass eigenvalues are

$$m_q = m_0, \quad M_i = k_{EW} \left(x_f^{(i)} + \frac{\hat{\chi}_i^2}{2} \delta \right), \quad M_{n_\infty+i} = k_{EW} \left(x_f^{(i)} - \frac{\hat{\chi}_i^2}{2} \delta \right), \tag{40}$$

and the elements of the orthonormal matrix $\mathcal{N}$ are parameterized by

$$\mathcal{N}_{(i',\,j')} \equiv \mathcal{N}_{(i',\,j')}^{(0)} + n_{(i',\,j')} \frac{\delta}{\sqrt{2}}, \tag{41}$$

where

$$
\begin{aligned}
&n_{(0,0)} \simeq 0, \\
&n_{(0,i)} \simeq 0, \quad n_{(j,0)} \simeq n_{(n_\infty+j,0)} \simeq \frac{\hat{\chi}_j^2}{x_f^{(j)}}, \\
&n_{(i,i)} \simeq n_{(n_\infty+j,j)} \simeq \frac{\hat{\chi}_i^2}{4x_f^{(i)}}, \\
&n_{(i,j)} \simeq n_{(n_\infty+i,j)} \simeq \frac{x_f^{(i)}\hat{\chi}_i\hat{\chi}_j}{\sqrt{2}(x_f^{(i)\,2} - x_f^{(j)\,2})} \quad (i \neq j)\,.
\end{aligned} \tag{42}
$$

IV. CONSTRAINTS FROM $B \to X_S\gamma$

Due to its important role in the current particle physics phenomenology, the rare decay of $B \to X_s\gamma$ has been extensively studied within and beyond the SM [21]. Moreover, this decay mode is sensitive to the top quark sector, appropriate to probe any new physics related with top quark. The inclusive decay $B \to X_s\gamma$ is approximated by the partonic decay $b \to s\gamma$ with the following equality:

$$
\frac{\Gamma(B \to X_s\gamma)}{\Gamma(B \to X_c e\bar{\nu}_e)} \simeq \frac{\Gamma(b \to s\gamma)}{\Gamma(b \to c e\bar{\nu}_e)} \equiv R_{\text{quark}}\,. \tag{43}
$$

With the NLO QCD corrections, R_{quark} is [22]

$$
R_{\text{quark}} = \frac{\lambda_t^2}{|V_{cb}|^2}\frac{6\alpha}{\pi f(z)}F(z)\left(|D(m_b)|^2 + A\right)\,, \tag{44}
$$

where $z \equiv m^2_{c,pole}/m^2_{b,pole}$ and

$$
\begin{aligned}
f(z) &= 1 - 8z^2 + 8z^3 - z^4 - 12z^2\ln z, \\
F(z) &= \frac{1}{\kappa(z)}\left(1 - \frac{8}{3}\frac{\alpha_2(m_b)}{\pi}\right)\,.
\end{aligned} \tag{45}
$$

The CKM factor is denoted by $\lambda_i \equiv V_{ib}^* V_{is} (i = u, c, t)$. The bremsstrahlung corrections and the necessary virtual corrections, in order to cancel the infrared divergence, are included in the term A [23].

In the RS-bulk SM the $b \to s\gamma$ decay obtains new contribution from the KK modes of the bulk W gauge boson and up-type quarks. If the bulk fermion masses are set to be universal for three generation quarks and the Yukawa interactions are ignored, the masses of the KK modes at a given level are the same for up, charm and top quarks. The unitarity condition of CKM matrix then cancels the contributions of the KK modes of u, c, and t quarks to FCNC, level by level. As the top quark Yukawa coupling turns on, the degeneracy in the masses of KK modes of up-type quarks is broken. New contributions to FCNC are developed.

Note that once the GIM mechanism does not work perfectly, the effects of the RS-bulk SM on the $b \to s\gamma$ decay have complicated and distinct features compared to those of other extra dimensional models. For instance, in the models with universal flat extra dimensions accessible to all the SM fields, the conservation of extra dimensional momentum leads to the so-called KK number conservation [8], which says that no single KK excited mode can be produced. This reduces the computation of its contribution to FCNC [9]. In the RS-bulk SM, however, non-trivial geometry does not respect this KK number conservation with which, e.g., $C^{ff'W}_{00n}$ would vanish for $n \neq 0$. Furthermore, in universal flat extra dimensions the bulk wave functions of bulk fermions are the same as those of bulk gauge bosons. The orthonormal conditions for the bulk wave functions simplify three-point couplings so that $C^{ff'W}_{0nm}(\text{FLAT}) = \delta_{nm}$. In the RS model, the bulk fermions have different bulk wave functions, sensitively dependent on the bulk fermion mass m_ψ. Non-trivial three-point couplings become involved.

Note that all the external particles are the SM particles, i.e., the zero modes of the bulk fields. Since $\chi^{(0)}_A$, the bulk wave function of the photon zero mode, is constant, the orthonormal conditions in Eqs. (7) and (13) imply

$$C^{WWA}_{nm0} = \delta_{nm}, \quad C^{f\bar{f}A}_{nm0} = \delta_{nm}. \tag{46}$$

Thus the contribution via the n-th up-type quark and the m-th W boson is the same as the SM result except for the internal mass and the additional three-point coupling.

With a given $W^{(l)}$, the contributions of all the KK modes of massless up and charm quarks to $(\lambda_u + \lambda_c)D$ in Eq. (44) are

$$(\lambda_u + \lambda_c) \sum_{\underline{i}=0}^{2n_\infty} \left(\sum_m C^{btW}_{0ml} \mathcal{N}^{(0)}_{(\underline{i},m)} \right)^2 D(x^u_{(\underline{i},l)}) = -\lambda_t \sum_{j=1}^{n_\infty} \left(C^{btW}_{0jl} \right)^2 D(x^u_{(\underline{i},l)}), \tag{47}$$

where $x^q_{(\underline{i},l)} \equiv \left(M_q^{(\underline{i})}/M_W^{(l)}\right)^2$. For the equality in Eq. (47), we have employed the unitarity condition of the CKM matrix ($\lambda_u + \lambda_c + \lambda_t = 0$) and the exact expressions for $\mathcal{N}^{(0)}$ in Eq. (39). In summary, the incorporation of all the KK modes of bulk gauge bosons and up-type quarks is achieved by the following replacement:

$$D(x^t_{(0,0)}) \Rightarrow D^{RS} \equiv \left[\sum_{l=0}^{n_\infty}\sum_{\underline{j}=0}^{2n_\infty}\left(\sum_{m=0}^{n_\infty} C^{btW}_{0ml}\mathcal{N}_{(\underline{j},m)}\right)^2 D(x^t_{(\underline{j},l)}) - \sum_{j,\,l=1}^{n_\infty}\left(C^{btW}_{0jl}\right)^2 D(x^{(0)}_{(j,l)})\right]. \quad (48)$$

In the limit where the elements of $\mathcal{M}_Y$ are much smaller than those of $\mathcal{M}_{KK}$, the substitution of Eqs. (40) and (42) into(48) yields, to leading order in δ,

$$\begin{aligned} D(x^t_{(0,0)}) \Rightarrow D^{RS} &\simeq D(x^t_{(0,0)}) \qquad (49)\\ &\quad + \sum_{l=1}^{n_\infty}\sum_{j=1}^{n_\infty}\left[\left\{\sum_{m=1}^{n_\infty} C^{btW}_{0ml}\left(\frac{\delta_{jm}}{\sqrt{2}} + \frac{n_{(j,m)}}{\sqrt{2}}\delta\right)\right\}^2 D\left(x^{(0)}_{(j,l)} + \hat{\chi}^2_j\, x^{(0)}_{(j,l)}\delta/x^{(j)}_f\right)\right.\\ &\qquad + \left\{\sum_{m=1}^{n_\infty} C^{btW}_{0ml}\left(-\frac{\delta_{jm}}{\sqrt{2}} + \frac{n_{(n_\infty+j,m)}}{\sqrt{2}}\delta\right)\right\}^2 D\left(x^{(0)}_{u(j,l)} - \hat{\chi}^2_j\, x^{(0)}_{u(j,l)}\delta/x^{(j)}_f\right)\\ &\qquad \left. -(C^{btW}_{0jl})^2 D\left(x^{(0)}_{(j,l)}\right)\right]\\ &= D(x^t_{(0,0)}) + \mathcal{O}(\delta^2)\,, \end{aligned}$$

where we have used the approximation

$$x^t_{(j,\,l)} \simeq x^{(0)}_{(j,\,l)}\left(1 + \hat{\chi}^2_j/x^{(j)}_f\right), \quad x^t_{(n_\infty+j,\,l)} \simeq x^{(0)}_{(j,\,l)}\left(1 - \hat{\chi}^2_j/x^{(j)}_f\right). \quad (50)$$

We conclude that in the limit where the Yukawa masses are smaller than the KK masses, cancellation occurs among leading contributions from the KK modes of W bosons and up-type quarks in the RS-bulk SM.

V. NUMERICAL RESULTS

All the phenomenological signatures of the RS-bulk SM depend on four parameters, $\Lambda_\pi(\equiv M_{Pl}\,e^{-kr_c\pi})$, $k_{EW}(\equiv k\,e^{-kr_c\pi})$, the Yukawa mass of top quark m_0, and $\nu\,(\equiv m_\psi/k)$. Λ_π characterizes the interaction of KK gravitons and the SM particles with the effective Lagrangian of $\mathcal{L}_{eff} = -(1/\Lambda_\pi)\,T^{\mu\nu}(x)\sum_{n=1}^{\infty} h^{(n)}_{\mu\nu}(x)$ [24]. Here $T^{\mu\nu}(x)$ is the energy momentum tensor. The ratio of k to M_{Pl} is assumed to be small since the five-dimensional curvature $R_5(= -20k^2)$ should satisfy $|R_5| < M_{Pl}^2$ for the self-consistency of the RS metric solution obtained from the leading term in the curvature. Thus we consider k_{EW} below 10 TeV, otherwise $\Lambda_\pi \gtrsim 100$ TeV, which is inappropriate as a solution to the gauge hierarchy problem. The Yukawa mass m_0 of top quark is of the order 100 GeV. And the parameter space of $\nu \in [-0.5, -0.3]$ is to be studied, as discussed in Sec. II B. For the $b \to s\gamma$ decay where charged currents are involved, three parameters of m_0, k_{EW} and ν determine the contributions completely.

First let us check whether the contributions to $b \to s\gamma$ converge or not, as we add more and more KK modes of W boson and up-type quarks. It is useful to introduce the cut-off on the number of the KK states, denoted by n_C, and check the sensitivity of D^{RS} in Eq. (48) to n_C. $D(m_b)$ is obtained by matching of Wilson coefficients at electroweak scale and the subsequent RGE running. For the SM contribution the matching is performed at the next-to-leading order (NLO) while the matching for the RS-bulk SM effects at leading order. Then D^{RS} gets the KK mode effects through C_7^{RS} in the leading order. With the formula of Eq. (48), therefore, we numerically compute C_7^{RS} as a function of n_C. In principle we should diagonalize an infinite dimensional matrix of top quark, $\mathcal{M}_{top}$, to obtain its mass eigenstates and mixing matrix. In reality of numerical calculation, however, this is not possible. Instead we take some large finite number n_∞, and truncate the contributions of KK modes at $n_C < n_\infty$. Figure 2 shows C_7^{RS} with the RS-bulk SM effects as a function of n_C for $kr_c = 11.5$, $\nu = -0.3$, $k_{EW} = 10$ TeV, and $m_0 = 200$ GeV. The dashed line is for $n_\infty = 50$, the dashed-dotted line for $n_\infty = 100$, and the dotted line for $n_\infty = 200$. Numerical diagonalization of a large dimensional matrix is performed by a FORTRAN package LAPACK [25]. The sudden drops as n_C approaches n_∞ are due to the edge effect from the introduction of finite dimensional matrix $\mathcal{M}_{top}$, as we observe that the falling slope becomes gentler as n_∞ increases. If n_∞ becomes really infinity the modified C_7^{RS} converges with increasing n_C. It has been shown that the RS-bulk SM effects on the precision electroweak observables [11], and on the anomalous magnetic moment of the muon [20] are also finite. In what follows we employ $n_\infty = 100$ and $n_C = 50$.

In Figs. 3, we present phenomenological constraints on (k_{EW}, m_0) plane for $\nu = -0.3$, and $\nu = -0.4$. First, the solid lines come from the observed top quark mass of 175 ± 5 GeV. For large k_{EW} of a few TeV, m_0 itself is the physical mass of the top quark, while low k_{EW} of the order 100 GeV allows considerably high m_0. A remarkable point is that the observed top quark mass alone can put a significant lower bound on k_{EW} such as $k_{EW} \gtrsim 210$ GeV for $\nu = -0.3$, and $k_{EW} \gtrsim 310$ GeV for $\nu = -0.4$. Second, the dashed lines correspond to the condition that the first excited KK mode of top quark, $M_1^{\rm top}$, be heavier than 1 TeV. This condition removes most of the parameter space with low k_{EW} and high m_0, and sets stronger bounds on k_{EW} such as $k_{EW} \gtrsim 500$ GeV for $\nu = -0.3$, and $k_{EW} \gtrsim 700$ GeV for $\nu = -0.4$. Finally, the dot-dashed lines are from 99% CL constraints allowed by experimental results of ${\rm Br}(B \to X_s + \gamma)$ [22]. With the constraint from the observed top quark mass, the $b \to s\gamma$ decay imposes the strongest lower bound on k_{EW} (shaded region): $k_{EW} \gtrsim 2$ TeV for $\nu = -0.3$, and $k_{EW} \gtrsim 3$ TeV for $\nu = -0.4$. As can be seen in Fig. 3, negatively larger parameter of ν has stronger bound from the observed $b \to s\gamma$ decay. This behavior is opposite to the collider signatures which receive smaller KK effects as ν decreases to negative values. Therefore, the constraint from the $b \to s\gamma$ decay on the RS-bulk SM can be complementary to the high energy collider observables.

VI. CONCLUSIONS

We have studied the top quark Kaluza-Klein mode mixing in the Randall-Sundrum scenario, where all the SM fields except for the Higgs field are placed in the bulk. This KK mode mixing occurs due to the Yukawa masses of the bulk fermion with the Higgs field confined to our brane. We have reviewed in detail the KK reduction of the bulk Dirac fermion field in the RS background. In order to obtain non-trivial solution of the bulk wave functions, a Dirac fermion with a definite hypercharge should possess both chiral states. That is, there must be additional right-handed $SU(2)$-doublet and left-handed $SU(2)$-singlet fermions which are to be assigned Z_2-odd symmetry to avoid their zero modes on our brane.

It is explicitly shown that the KK mass terms are between the left- and right-handed chiral states of a given representation while Yukawa couplings relate the $SU(2)$-doublet and $SU(2)$-singlet. For the top quark of non-negligible Yukawa mass, this mismatch between KK mass matrix and Yukawa mass matrix generates the mixing among the KK modes of the top quark.

One of the immediate problems of this top quark KK mode mixing is FCNC. In the literature, the assumption of universal bulk fermion mass for three generation quarks has been regarded useful to avoid FCNC because without Yukawa couplings the KK mass spectra become the same for up, charm and top quarks. With the assumption of minimal flavor mixing, the GIM cancellation due to the unitarity of CKM matrix works KK-level by level. As the top quark Yukawa coupling comes in, breaking of KK mass degeneracy causes small but unavoidable contributions to FCNC. In the parameter space where the top quark Yukawa masses are smaller than the KK masses, perturbative approaches are employed to show that their contribution to the rare decay $b \to s\gamma$ vanishes to leading order. For other parts of the parameter space, it is demonstrated that the contribution of the RS-bulk SM can be numerically computed with high reliability. The current measurements of $\mathrm{Br}(B \to X_s + \gamma)$ with the observed top quark mass lead to quite strong bounds on k_{EW} such as $k_{EW} \gtrsim 2$ TeV for $\nu = -0.3$ and $k_{EW} \gtrsim 3$ TeV for $\nu = -0.4$ at 99% CL. Moreover, the dependence of the $b \to s\gamma$ contribution on the parameter ν is contrary to that of the collider signatures. For example, $\nu = -0.3$ ($\nu = -0.4$) case would leave larger (smaller) collider signatures but smaller (larger) $b \to s\gamma$ contribution. Thus, high energy collider and rare B decays can play complementary roles to probe the effects of the RS-bulk SM.

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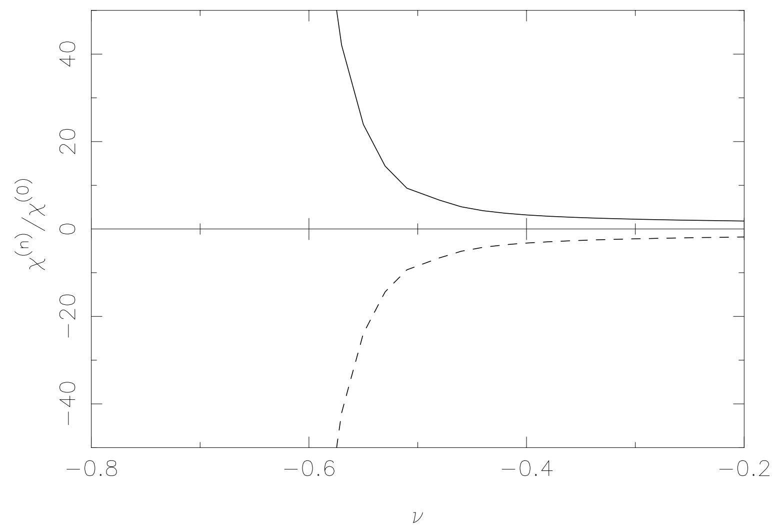


FIG. 1. $\hat{\chi}^{(n)} \equiv \chi^{(n)}/\chi^{(0)}$ as a function of ν with $kr_c = 11.5$. The solid line is for odd modes ($n = 1, 3, 5, ...$) and the dotted line for even modes ($n = 2, 4, 6, ...$).

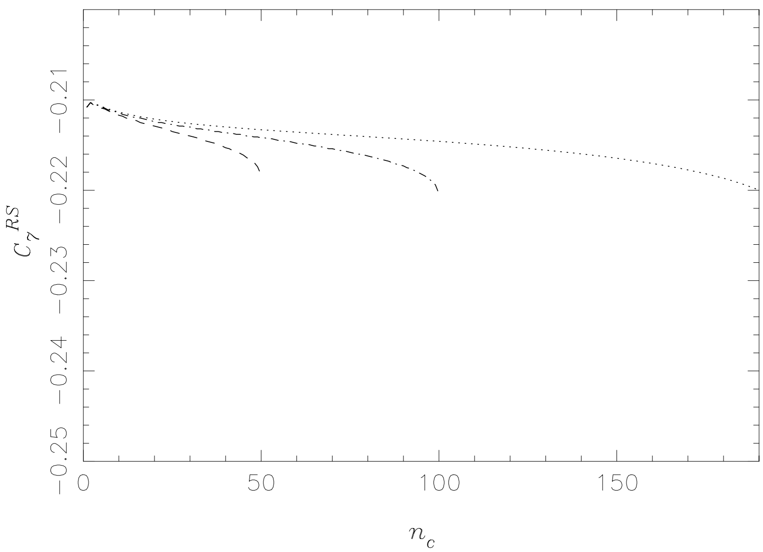
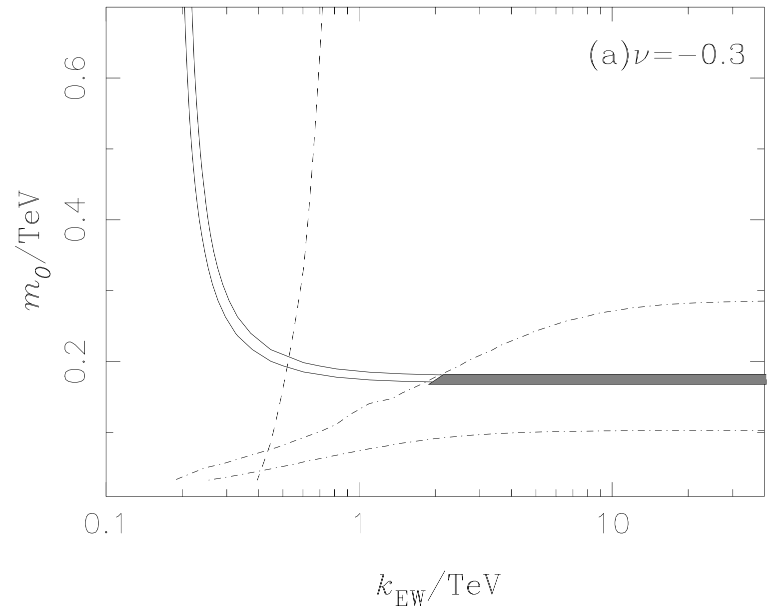


FIG. 2. C_7^{RS} as a function of n_c, with $k_{EW} = 10$ TeV and $m_0 = 200$ GeV. The dashed line is for $n_\infty = 50$, the dashed-dotted line for $n_\infty = 100$, and the dotted line for $n_\infty = 200$.



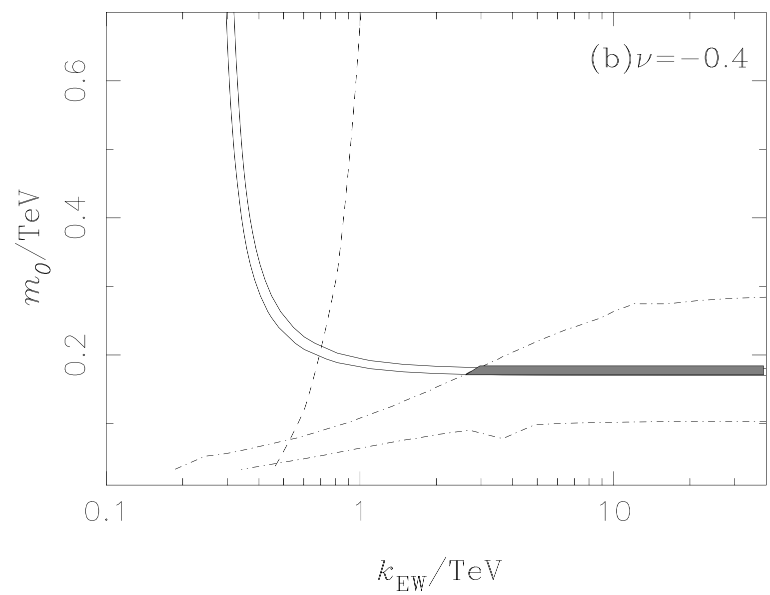


FIG. 3. The (k_{EW}, m_0) parameter for (a) $\nu = -0.3$ and (b) $\nu = -0.4$. We take $kr_c = 11.5$, $n_\infty = 100$ and $n_c = 50$. The solid lines are from the top mass constraints. The dashed line is from $M_1^{\rm top} > 1$ TeV, and the dash-dotted lines from the measurement of $b \to s\gamma$ at 95%CL. The shaded area is the final allowed region.